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Stochastic Growth Processes in Large Finnish Companies: Test of Gibrat's Law of Proportionate Effect

ABSTRACT

The purpose of the study is to analyse the growth processes, as well as the relationship between size and growth, in large Finnish firms in 1987–1995. The study concentrates on stochastic growth models and on the test of Gibrat's law of proportionate effect. This law assumes that growth is a random process and independent of the size of the firm. If Gibrat's law holds, there is no optimum size for the firm with respect to growth. Several modifications of Gibrat's law are briefly discussed to form a basis for the empirical study. The data are taken from the ETLA data base including financial statements from the 500 largest firms in Finland. Due to the restrictions set on the stationarity of the data, only 157 firms out of 500 are accepted for the sample. The size of the firm is measured by nominal net sales. Markov transition matrices and regression analysis are used as the main methods of testing Gibrat's law. The concentration of the size distribution is analysed by the Hirschman-Herfindahl index, Gini coefficient, and concentration ratios. The empirical results show that there are no changes in concentration in the research period in spite of the radical changes in business cycles. There are no

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large differences in growth distribution between size classes. However, in the largest size class the growth seems to be slower than in smaller classes. Moreover, the firms in the smallest size class tend to grow rather fast. These two phenomena contradict with Gibrat's law and may have caused the concentration to stay stable over time. A negative relationship is observed between the growth and size. There is also a statistically significant negative relationship between the four-year growth rates. Thus, also the results on the persistence of growth violate Gibrat's law.

Key words: *Gibrat's law – stochastic growth processes – large firms – concentration*

1. INTRODUCTION

There is a universal tendency for economic variables to be distributed according to positively skewed distributions. This also holds for the size distributions of business firms (for the classic study see Hart and Prais 1956). The form of the size distribution refers to the concentration of economic activity and is based on the growth processes going on in the firms. Therefore, it can be said that concentration and growth are the static and dynamic expression of the same very important economic phenomenon. This means that if we want to develop an explanation for concentration in economics, we must analyse the growth processes in business firms. There are several different classes of growth theories used to explain concentration in economics (see Evans 1987 and Sapienza et al. 1997). First, many of the growth models are based in some way on *Gibrat's law of proportionate effect,* according to which growth is stochastic and thus independent of the size of the firm. For example, Simon and Bonini (1958) assume that Gibrat's law holds for firms above the minimum efficient size level. Lucas (1978) presented an influential model of the size distribution of firms that assumes Gibrat's law in order to prove the existence and uniqueness of an equilibrium. The model presented by Lucas in 1967 on capital adjustment implies that the time series of firm employment, capital, and output conforms to Gibrat's law (see Lucas 1967). Jovanovic (1982) developed special cases of his model of firm learning in which the law holds at the limit for mature firms or for firms entered the industry at the same time.

Second, there are a number of growth models conceptualizing the growth of the firm as a sequence of progressive stages (Sapienza et al 1997). In these models, firms are expected to go through some sequence of stages (see Greiner 1972, Churchill and Lewis 1982, Kazanjian 1988, and Kazanjian and Drazin 1990). For example, Greiner (1972) introduced a general growth model based on five stages of growth and links the speed of growth to the industry's growth rate and its level of profitability. Churchill and Lewis (1982) focused on the early stages after foundation and considered also periods of nongrowth perceived as disengagement or failure. Later Scott and Bruce (1987) linked this framework to managerial, organizational, and industry issues. In the Kazanjian (1988) model each stage of growth is associated with dominant problems facing the firm arising from scaling up production and building sales capacity to gain market share. Third, the resource-based models of firm growth emphasize the role of resources as the ultimate determinant of profit generation potential (see Sapienza et al. 1997). For example, in the Penrose (1959) model the growth is stimulated by the need to exploit the resources of the firm maximally. Garnsey (1996) builds his model directly on the Penrosian theory adding new insight by drawing on systems thinking.

The growth model classes roughly outlined above differ strongly from each other. The earliest of the stochastic theories are very useful in that they are only dealing with size and growth using stochastic growth processes to explain the size distribution. Therefore they are very handy in empirical research and require only a few variables to be estimated and tested. *For these reasons, the present paper concentrates on growth as a stochastic process and largely rests on the test of Gibrat's law.* There is also some previous empirical evidence on these matters (see Evans 1987). For example, the studies of large firms by Kumar (1985), Evans (1986), and Hall (1987) found that Gibrat's law fails for several different measures of firm size so that firm growth decreased with firm size. In Finland, Hajba (1978) analysed the growth of the 400 largest firms in the years 1968–1973. She also found a negative relationship between size and growth although the coefficients of multiple determination in her regression equations were low supporting the stochasticity of growth. When size was measured by net sales, the average coefficient of multiple determination was 15.2% (Hajba 1978: 176). Neilimo and Pekkanen (1996) analysed the growth of nine large companies in the years 1986–1993. They showed that in most cases the growth rates of firms were stable, which conforms to the hypothesis that the growth rate is rather independent of the size of the firm.

The purpose of the study is to analyse the growth process, as well as the relationship between size and growth, in large Finnish firms in 1987–1995. This time period is very interesting because of radical changes in business cycles. Table 1 presents some statistics to demonstrate the environment faced by Finnish companies during the research period. This table shows that after rapid expansion of economic activity during 1987–1990, a deep depression started in 1991. For example, in construction the volume of production in the years 1993–95 was only 64–65% of the level in 1990. The number of unemployed rose from 88 000 in 1990 to 456 000 in 1994. The number of bankruptcies more than doubled during 1990–1992. The Helibor interest rate rose from 10 percent in 1987 to 14.0 percent in 1990 and again declined to 5.8 percent in 1995. Total investments were about FIM 139 billion in 1990 but declined rapidly and were only FIM 83 billion in 1995. Therefore, the research period is very interesting from the viewpoint of concentration and growth of business companies. Did the radical changes in the economy affect the growth process in large firms? Under these special circum-

TABLE 1. Economic environment of Finnish business companies in 1987–1995

Panel 1. Economic time series (source: Statistical Yearbook of Finland 1996)

	Year:								
Time series:	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**
Index of wage and salary earnings (1985=100)	114.5	124.8	135.8	148.3	157.7	160.7	161.9	165.1	172.8
Gross domestic product at 1990 prices, billions of FIM	246.2	258.8	269.9	269.8	260.0	247.4	240.2	244.8	254.1
Investments at current prices, billions of FIM	92.5	109.3	136.1	139.1	110.1	88.0	71.2	74.2	83.3
Helibor interest rate (3 months)	10.0	10.0	12.5	14.0	13.1	13.2	7.7	5.3	5.8
Employed, thousands	2423	2431	2470	2467	2340	2170	2041	2024	2068
Unemployed, thousands	130	116	89	88	193	328	444	456	430
Unemployment rate, %	5.1	4.5	3.5	3.4	7.6	13.1	17.9	18.4	17.2
Bankruptcy petitions filed	2816	2547	2717	3588	6253	7355	6768	5502	4654

Panel 2. Volume index of production (1990=100) by industries (source: Domestic Databases of ETLA)

	1987	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**
Industry	93.0	96.8	100.7	100	89.1	90.9	95.8	107.8	118.3
Construction	82.2	89.7	102.4	100	88.4	75.2	64.4	63.1	65.5
Wholesale and retail trade	89.8	94.9	102.7	100	86.8	75.8	71.5	75.3	77.9
Hotels and restaurants	87.8	91.9	97.7	100	92.1	85.2	81.1	84.7	85.4
Transportation (*)	81.0	86.1	94.3	100	96.3	96.2	98.6	103	107

*** Source: Statistics Finland: National Accounts**

stances, did Gibrat's law hold? What happened to the rate of concentration of size distribution due to the growth of firms? These are questions to be answered in this study. The paper is organized as follows. The background of the study was presented in the first introductory section. The second section summarizes the main stochastic processes based on Gibrat's law. The data and methods are presented in the third section and the fourth section includes the empirical results on size concentration and growth. Finally, the last section summarizes the main findings of the study.

2. STOCHASTIC GROWTH MODELS BASED ON GIBRAT'S LAW

Gibrat based his law of proportionate effect initially on Laplace's law of constant effect, which can be applied only to symmetrical size distributions (Gibrat 1931: 62–90). The law of proportionate effect can be expressed by the fact that the logarithms of the size variable are distributed by the law of constant effect. The stochastic process of Gibrat's law is presented generally as a random walk process which takes place on a logarithmic scale (see for example Kalecki 1945). *In a process of growth the law of proportionate effect means that equal proportionate increments have the same chance of occurring in a given time-interval whatever size happens to have been reached, that is growth in proportion to size is a random variable with a given distribution which is considered constant in time.* In the growth process of firms Gibrat's stochastic law means that the logarithm of the size results from the addition of many small random variables having identical distributions with mean u and variance q^2 to the logarithm of the original size. By the central limit theory the logarithmic size is asymptotically normally distributed with mean $n \cdot u$ and variance $n \cdot q^2$, where n is the number of periods. Thus the size distribution of firms at time n will be approximately lognormal.

The general lognormal distribution law means that the logarithm of size is distributed according to the normal distribution. In the case of lognormal distribution the variance of logarithmic size can be expressed as follows:

(1) $$q^2 = \log[1 + (v^2 / m^2)]$$

where v is the standard deviation of size and m the mean size. The expression (1) shows that the variance of logarithmic size will increase whenever the coefficient of variation of the size increases. Kalecki (1945), for example, uses the variance of logarithmic size as a measure of concentration. Since Gibrat's law assumes that the variance of logarithmic size is increasing all the time with the number of periods of growth, it assumes that concentration in the size distribution increases continuously. Summarizing, the appearing of the law of proportionate effect in the growth process means the following. First, there is no optimum size for the firm, that is, growth is a stochastic process. Second, there is no serial correlation in growth rates, that is, there is no continuity in the growth pattern. Third, the distribution of size tends asymptotically to the lognormal distribution and the variance of logarithmic size increases continuously.

In the firm populations where the concentration rate is very high it seems to be reasonable to expect that the rate will no longer increase because of the diseconomies of high concentration and the ensuing restricted growth process in the largest firms. In that kind of situation it is possible to apply Kalecki's random walk model (Kalecki 1945), where the variance of the logarithmic size remains constant in spite of random growth, to explain the concentration process. In this stochastic model *a negative linear correlation between the logarithm of random increment and the logarithm of size is assumed to hold* instead of the independence assumption of Gibrat's stochastic process. However, the other assumptions of Kalecki's model are identical with those in Gibrat's law.

Champernowne (1953) has presented a stochastic Markov chain model that leads according to its ergodic feature to the steady state distribution conforming to the exact Pareto distribution. The Pareto distribution is defined as follows

(2) $Q(x) = c\, x^{-p} \quad x > 0,\ c > 0,\ p > 0$

where p is the Pareto coefficient, c a constant, and Q(x) the function which gives the number of the firms whose size is greater than x. If the Pareto coefficient is close to unity, the size distribution conforms to the rank-size rule. In Champernowne's process Gibrat's law of proportionate effect is valid, but as a stability condition to offset the tendency for diffusion, he assumes that *the mathematical expectation of the random steps in the transition matrix of periodic size is negative.*

The assumption of the negative expectation is suitable for populations of impedent growth, but in fact Champernowne presented his stochastic process to explain the genesis of the Pareto tail of income distribution, that is, it is valid only above a lower limit of size. In that case the Pareto distribution function can be presented in the following form

(3) $F(x) = 1 - (x_0 / x)^{p} \quad x > x_0$

where x_0 is the lower limit of size. The differentation of function (3) with respect to x leads to the following Pareto density function

(4) $f(x) = (p / x_0)\, (x_0 / x)^{p+1}$

This function is a monotonically decreasing function of x, that is, the lower limit of size is the mode of the distribution.

The Pareto coefficient is often used as a measure of inequality, that is, relative concentration. Similar to the variance of the logarithmic size, the Pareto coefficient is in a close relationship with the coefficient of variation. If the Pareto coefficient is greater than 2, that is, if the distribution has a finite standard deviation, the following relationships hold between the arithmetic mean, the variance of size, and the Pareto coefficient

(5a) $m = p/(p-1)$

(5b) $v^2 = p/[(p-2)\,(p-1)^2]$

The solution of the set of equations (5) makes it possible to present the Pareto coefficient in the following form

(6) $p = 1 + [1 + m^2 / v^2]^{0.5}$

which is comparable to (1). Thus, the Pareto coefficient of the size distribution decreases when the coefficient of variation of the distribution increases.

The stochastic processes roughly outlined above are all *stationary* and do not include the birth-and-death process of firms. Because the empirical part of this study does not include the birth-and-death process, the idea of the nonstationary models is only shortly reviewed. Most of the birth-and-death processes based on Gibrat's law generate as the result of the growth process a skewed distribution, which conforms only in the tail to the Pareto distribution. Simon (1955) calls this kind of family of skewed distributions the Yule distributions. The tail of these distributions conforms to the following function

(7) $f(x) = (a / x^k)\, b^x$

where a, b, and k are constants. Simon argues that b is so close to unity that in the first approximation the final factor has a significant effect on f(x) only for very large values of x. He also says that the exponent k is greater than unity but smaller than 2. Simon presented a stochastic model that jointly by Gibrat's law and an assumed birth process generated the Yule distribution (7). Simon assumes that there is a constant birth rate of new firms in the smallest size class. Simon and Bonini (1958) argued that lognormality is only a special case of the Yule distribution which appears when the birth-and-death process has not been considered. Ijiri and Simon (1977) presented a modification of Simon's model in which serial correlation between the periodic growth rates is allowed. They showed by Monte Carlo simulations that also this modification will generate a Yule type of distribution, although a perfect steady state was not reached.

The short discussion of the models above can be summarized as follows. If the growth of firms is stationary and does not include the birth-and-death process, Gibrat's law generates a lognormal distribution and the variance of logarithmic size will continuously increase over time (pure Gibrat's law). However, if there is a linear negative dependence between the logarithms of growth and size, Gibrat's law will generate a lognormal distributíon but with a constant variance of logarithmic size (Kalecki's model). If the mathematical expectation of increments in the stochastic transition matrix is negative, Gibrat's law generates a Pareto distribution (Champernowne's model). The incorporation of the birth-and-death process to Gibrat's law leads to a Yule distribution when there is a constant rate of birth (Simon's model) or serial correlation between periodic growth rates (Ijiri-Simon's model).

3. THE DATA AND METHODS OF THE STUDY

The empirical data for the present study have been taken from the ETLA data base including financial statements from the 500 largest firms in Finland. However, we set two restrictions for the choice of the firms to the empirical study. First, while the research period covers the years

1987–1995, it was required that the financial statements for the firm to be chosen were also available for the whole period. Second, all the firms that were involved in large mergers during the years 1987–1995, were excluded from the sample. The first restriction was set because we wanted to keep the growth process stationary in the way that the observations (firms) in each year were the same. For the sake of this restriction the results of different years are strictly comparable to each other. The second restriction of the exclusion of mergers is based on the aim to focus on *the internal growth processes,* thus excluding external growth. When considering the stochastic processes based on Gibrat's law, it is reasonable to concentrate on internal growth only. The restrictions meant that only 157 firms out of 500 were accepted for the sample.

For simplicity, the research period was split into only two sub-periods, 1987–1991 and 1991–1995 so that four-year growth rates are considered as such and also as transformed to annual equivalents. For the same reason only one measure of size, *nominal net sales,* is considered. Net sales is a reliable indicator of economic activity, and is, as a flow variable, more sensitive to stochastic changes than a stock variable (for example, total assets). Net sales was used instead of value added because the latter obtained a number of negative values in the data. Table 2 shows the industrial classification and the size of the 157 sample firms. The distribution of the firms among industries is very diversified. Wholesaling, retailing, as well as food, metal, and multi-industries have the highest frequencies in the sample. The median size measured by net sales in 1987 is about FIM 450 million and in 1995 about FIM 670 million. Thefore, the typical firm in the sample is middle-sized according to Finnish standards. However, besides the large firms at the top of the size distribution, the sample also includes firms with net sales only a bit over FIM 100 million. *To summarize, the data consist of large and middle-sized Finnish firms with a high variety in size.* Note that the large difference between the mean and the median values in Table 2 implicitly refers to a very skewed size distribution, and, at the same time, to a high rate of concentration.

The size distribution of net sales is analysed in this study in several different ways. First, the frequency distributions of size are presented. Second, a set of familiar statistical measures of distribution (mean, standard deviation, skewness, and kurtosis) are calculated for both the net sales and the logarithmic net sales distribution. Third, the compatibility of the size distributions with normal, lognormal, and Pareto distributions are evaluated. The normality and lognormality of the distributions are tested by the Jarque-Bera test statistic (see Jarque-Bera 1980). The Pareto law is tested as follows. The Pareto coefficient is calculated on the basis of equation (6) by size class, using arithmetic mean and standard deviation of the tail. If the Pareto law is valid for a certain lower limit of size, the coefficients will be *identical* in each class upwards. Therefore, the stability of the coefficient gives us a simple test of the existence of a

TABLE 2. Industry and and size of sample companies

Panel 1. Industrial classification

Industry:	Percent:	Frequency:
Industry:		
Food manufacturing	12.1	19
Metal industry	11.5	18
Forest industry	3.2	5
Electronic industry	7.0	11
Chemical industry	3.2	5
Graphic industry	9.6	15
Multi-industry	10.8	17
Other	2.5	4
Service	4.4	7
Construction	5.7	9
Transportation	1.9	3
Energy production	1.9	3
Wholesaling	14.0	22
Retailing	12.1	19
TOTAL	100.0	157

Panel 2. Net sales in the years 1987, 1991 and 1995 (in millions of FIM)

Statistic:	1987	1991	1995
Mean	1576.668	2257.433	2539.883
95% percentile	7968	10599	12108
Upper quartile	1035	1548	1739
Median	452	621	672
Lower quartile	223	312	329
5% percentile	126	183	180

Pareto tail. Third, three kinds of concentration measures are used, that is, concentration ratios, the Hirschman-Herfindahl index, and the Gini coefficient, to evaluate the concentration of net sales from different perspectives (for the measures see Bailey and Boyle 1971, Hall and Tidemann 1967, Kilpatric 1967, and Marfels 1971). These measures are briefly reviewed in the following text.

The most popular measure of concentration may be the concentration ratio, which here measures the relative share of net sales for the m largest firms. Therefore the mth ratio of concentration is defined as

(8) $$Cr_m = \sum_{i=1}^{m} P_i$$

where P_i is the share of net sales for the ith largest firm. This ratio is very handy but is only based on one point of the size distribution. The rest of the measures are based on the whole distribution.

The use of the Hirschman-Herfindahl index (H-index) as a measure of concentration is supported by several researchers (see for example Marfels 1971). This index can be presented in the following way

$$(9) \qquad H = \sum_{i=1}^{n} P_i^2$$

where n is the number of firms in the population. Because of the squared form of the measure, it gives less weight to the firm, the smaller the size. The higher H is, the higher the rate of concentration.

The Gini coefficient is based on the Lorenz curve of the size distribution. Statistically the Lorenz curve presents the relationship between the distribution function and the first-moment distribution function. The Gini coefficient is the ratio of the area between the Lorenz curve and the diagonal line to the area of the triangle under the diagonal line. Technically the Gini coefficient can be presented as

$$(10) \qquad G = 1 - 2\,L$$

where L is the area under the Lorenz curve. The values of the coefficient vary from zero (equal distribution) to unity (perfect concentration). Note that actually G is a measure of inequality. This means that it is *independent* of the number of firms and is, in that sense, unaffected by an important factor of population structure. In this study this feature does not, however, affect the comparability of the results because the number of firms is kept constant over time.

Gibrat's law will be tested in several ways analysing the growth process of the firms. First, Markov transition probability matrices are calculated using a logarithmic scale for the size. If Gibrat's law holds, the probability distributions should be identical in each row of the matrices. These matrices also give an indication of the negative expectation of growth. Second, the statistical distributions of growth are evaluated by size class to check the same things. Third, the relationship between size and growth will be analysed by the regression analysis. Let us present the relationship between the size in period t and the size in period t–1 as follows

$$(11) \qquad x(t) = a\,x(t-1)^b$$

where a and b are constants. When dividing both sides of (11) by x(t–1) we get

$$(12) \qquad x(t) / x(t-1) = 1+g = a\,x(t-1)^{(b-1)}$$

where g is the growth rate. If Gibrat's law holds and the growth distribution is identical with respect to size, the constant b does not significantly differ from unity. If, however, the constant b is greater (less) than unity, the growth rate is an increasing (decreasing) function of size. The constant b is in this study calculated by taking logarithms of both sides of (11) and applying regression analysis (OLS). The normality of error terms is tested by the Jarque-Bera statistic, which is also used in the regression analyses below.

Fourth, the assumption of a negative linear relationship between the logarithm of growth, i.e. log(1+g) or log(x(t)/x(t–1)), and the logarithm of size, or log x(t–1), is also explicitly evaluated by the Pearson correlation coefficient. Implicitly, this is equivalent to testing the hypothesis b=1 in the regression (12) above. Fifth, the persistence (serial correlation) of growth was evaluated firstly by calculating the transition probability matrice for the growth rates in the two periods considered and secondly by explaining the growth rate in the latter period by the one in the former period using regression analysis (OLS). Finally, the stochasticity of growth processes is evaluated by analysing the relation of growth to the return on investment ratio and to debt-to-assets ratio by means of the regression analysis (OLS) and cross-tabulation. Neilimo and Pekkanen (1996) found, analysing nine large company cases, that there is a relationship between these variables. However, they observed that there were no relationship between liquidity ratios and growth. In this study the return on investment ratio is calculated using net profit after interest expenses as the nominator of the ratio because it shows the profitability available for growth, better than the profit before interest. The financial statements in the ETLA data base are adjusted according to the recommendations given by the Committee of Corporate Analysis (YTN) so that the ratios are reliable measures. Most of the statistical estimations are made by the SAS statistical package. However, also Microsoft Excel was used, for example, to calculate the Jarque-Bera test statistic and concentration measures.

4. THE EMPIRICAL RESULTS OF THE STUDY

Table 3 presents the distributions for the net sales in years 1987, 1991, and 1995 (panel 1) and a set of statistical measures for these size distributions (panel 2). This table shows that the distributions have remained, roughly speaking, similar over the years although they have moved upwards for the sake of positive growth in the population. The relation of the standard deviation to the mean (the coefficient of variation) is in each year about 2.3 and also the standard deviation of the logarithmic net sales has been very stable over time. All these statistics indicate that there may not have happened any large changes in concentration either. However, the distributions are very skewed and they do not conform to the normality or lognormality at all as shown by the Kolmogorov D test. Appendix 1 shows the cumulative net sales distribu-

TABLE 3. Size distributions (net sales) in the years 1987, 1991, and 1995 (in millions of FIM)

Panel 1. Frequencies

	1987		1991		1995	
Upper limit:	**Frequency**	**Percent**	**Frequency**	**Percent**	**Frequency**	**Percent**
200	**34**	**21.7**	**16**	**10.2**	**13**	**8.3**
400	**40**	**25.5**	**36**	**22.9**	**40**	**25.5**
800	**38**	**24.2**	**42**	**26.8**	**36**	**22.9**
1600	**16**	**10.2**	**26**	**16.6**	**26**	**16.6**
3200	**10**	**6.4**	**12**	**7.6**	**15**	**9.6**
6400	**9**	**5.7**	**11**	**7.0**	**12**	**7.6**
12800	**7**	**4.5**	**9**	**5.7**	**9**	**5.7**
25600	**3**	**1.9**	**3**	**1.9**	**3**	**1.9**
51200	**0**	**0**	**2**	**1.3**	**3**	**1.9**
TOTAL	**157**	**100.0**	**157**	**100.0**	**157**	**100.0**

Panel 2. Size statistics

Statistic:	**1987**	**1991**	**1995**
Net sales:			
Mean	**1576.668**	**2257.433**	**2539.883**
Standard deviation	**3591.188**	**5237.603**	**5842.115**
Skewness	**4.610**	**5.439**	**4.980**
Kurtosis	**24.394**	**37.524**	**30.764**
Jarque-Bera statistic	**4448.828**	**9985.079**	**6840.132**
Probability level	**0.0001**	**0.0001**	**0.0001**
Logarithmic net sales:			
Mean	**6.324**	**6.688**	**6.773**
Standard deviation	**1.259**	**1.256**	**1.286**
Skewness	**0.944**	**0.977**	**0.932**
Kurtosis	**0.558**	**0.408**	**0.305**
Jarque-Bera statistic	**25.3549**	**26.0658**	**23.3375**
Probability level	**0.0001**	**0.0001**	**0.0001**

tions as well as the theoretical normal and lognormal distributions. The actual distribution is clearly too skewed to conform even to the lognormal distribution. This appendix also shows the estimates for the Pareto coefficient by size class. For the lower size classes as lower limits the coefficients are in each year about 1.9 to 2.0. However, the estimates are not stable with respect to size classes. This instability may indicate that there is not a Pareto tail in the size distributions. Table 4 shows the values for the concentration measures which support the interpretation above. The Gini coefficients, Hirschman-Herfindahl indices, and the concentration ratios are almost at identical levels for the years 1987, 1991, 1995. The concentration ratios for each m (rank) are so close to each other that the Lorenz curves based on this data were graphically almost identical. *Thus, we conclude that the growth processes in large Finn-*

TABLE 4. Concentration of sales in 1987, 1991 and 1995

Panel 1. Concentration measures

Measure:	1987	1991	1995
Gini Coefficient	0.7266	0.7248	0.7317
Hirschman-Herfindahl Index	0.0392	0.0404	0.0399

Panel 2. Concentration ratios

		Percent of net sales:		
Rank in size order:	Percent of frequency:	1987	1991	1995
150	95.54	99.71	99.68	99.71
140	89.17	99.17	99.14	99.21
130	82.80	98.53	98.51	98.60
120	76.43	97.76	97.76	97.89
110	70.06	96.83	96.85	97.05
100	63.69	95.70	95.74	96.07
90	57.32	94.44	94.43	94.91
80	50.96	92.89	92.85	93.33
70	44.59	90.91	90.97	91.54
60	38.22	88.63	88.79	89.39
50	31.85	85.95	86.18	86.64
40	25.48	82.52	82.82	83.08
30	19.11	77.51	77.69	77.61
20	12.74	69.58	68.99	69.10
10	6.37	52.36	50.62	52.31
9	5.73	49.69	47.78	49.68
8	5.10	46.80	44.93	46.87
7	4.46	43.58	41.94	43.84
6	3.82	40.34	38.88	40.74
5	3.18	36.89	35.69	36.55
4	2.55	32.12	31.87	31.82
3	1.91	27.28	27.44	26.54
2	1.27	20.03	21.19	19.91
1	0.64	10.19	13.16	12.34

ish firms in 1987–1995 have operated in such a way that the concentration of net sales has been extremely stable.

The results above may indicate that Gibrat's law does not hold in its pure form in the growth processes. Table 5 shows the Markov transition matrices for the periods 1987–1991 and 1991–1995. This table shows that the probability distributions by size class are not identical as assumed by Gibrat's law. However, there are no large differences either. These distributions can be interpreted so that smaller firms may have a bit better probabilities for positive growth than the larger ones and that the variations of growth in smaller firms are larger, too.

TABLE 5. Markov transition probability matrices

Panel 1. Transition from 1987 to 1991

Size class in 1987:	Size class in 1991:									
	1	2	3	4	5	6	7	8	9	Total
1	44.1	50.0	5.9							100
2	2.5	47.5	45.0	5.0						100
3			52.6	44.8	2.6					100
4			12.5	43.8	37.5	6.2				100
5					60.0	40.0				100
6						55.6	44.4			100
7							71.4	28.6		100
8								33.3	66.7	100
9										

Panel 2. Transition from 1991 to 1995

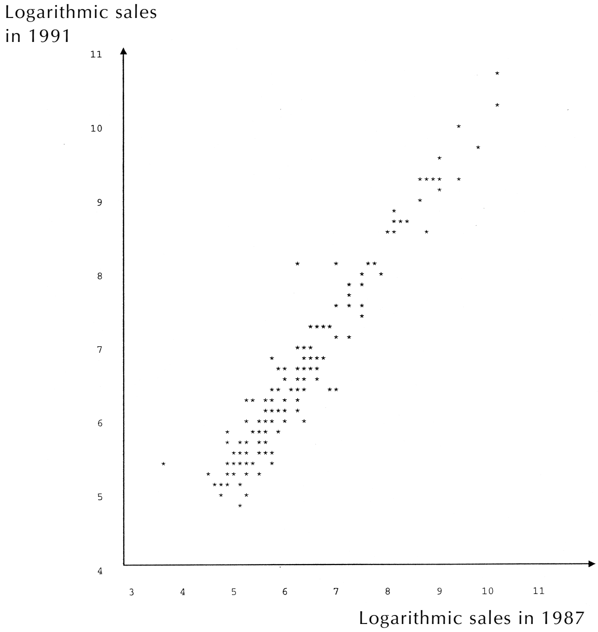
Size class in 1991:	Size class in 1995:									
	1	2	3	4	5	6	7	8	9	Total
1	37.5	56.3	6.2							100
2	16.7	69.4	13.9							100
3	2.4	14.3	52.4	28.6	2.4					100
4			30.8	50.0	15.4	3.8				100
5				8.3	66.7	25.0				100
6					18.2	54.5	27.3			100
7						11.1	66.7	22.2		100
8						33.3		33.3	33.3	100
9									100	100

Figure 1 shows the same information graphically. This figure includes the scatter diagrams between the logarithmic sales in successive periods. The distributions are almost homoskedastic. In the latter period the growth has been slower so that the scatter diagram is not as vertical as in the former period. However, there is more variation in growth rates in the latter period. Table 6 presents statistics of the annual growth rates by size class. These statistics support the conclusion above. There are no large differences in growth distribution between size classes. However, in the largest size class the growth seems to be slower than in smaller classes. Moreover, the firms in the smallest size class tend to grow rather fast. *These two phenomena contradict with Gibrat's law and may have made it possible for concentration to stay stable over time.*

Table 7 presents the results of the regression analysis for the successive logarithmic net sales. This regression analysis conforms to the graphic analysis of Figure 1. The linear relation-

FIGURE 1. Scatter diagrams between logarithmic sales figures

Panel 1. Logarithmic sales in 1987 and 1991



Panel 2. Logarithmic sales in 1991 and 1995

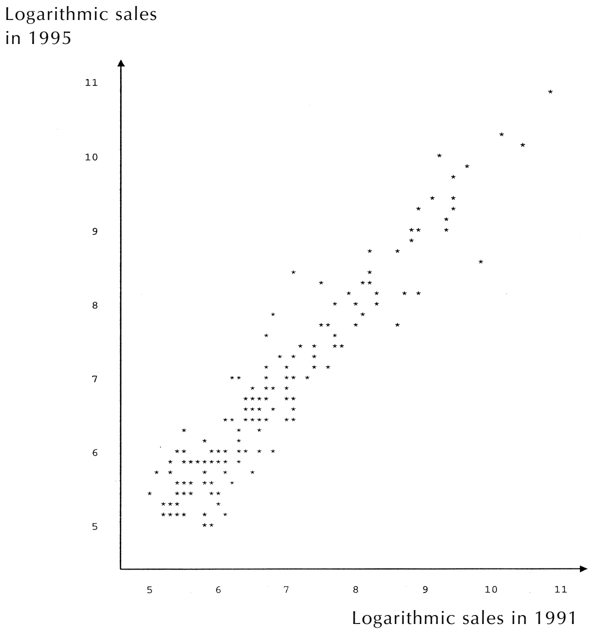


TABLE 6. Distribution of annual growth rate in net sales by size class

Panel 1. Annual growth rate in 1987–1991

Size class in 1987:	Mean	Upper quartile	Median	Lower quartile
1	12.78	15.88	11.08	7.31
2	8.91	13.79	8.44	3.92
3	10.12	13.69	9.26	4.65
4	7.44	14.88	6.94	0.78
5	8.23	13.09	9.66	2.97
6	11.72	16.02	15.35	8.86
7	8.76	14.21	11.16	3.88
8	5.77	16.59	3.99	–3.27

Panel 2. Annual growth rate in 1991–1995

Size class in 1991:	Mean	Upper quartile	Median	Lower quartile
1	7.37	14.69	5.80	1.36
2	0.92	5.69	0.27	–3.19
3	2.96	8.56	2.09	–2.65
4	2.65	6.45	0.35	–5.56
5	2.50	8.02	1.94	–4.50
6	1.78	8.19	3.39	–4.92
7	3.15	9.98	3.30	–1.33
8	–2.09	8.63	8.06	–22.95
9	–0.25	1.35	–0.24	–1.85

ship between the logarithmic sizes is statistically very significant and the coefficients of multiple determination exceed 90% in both periods. The estimates for the coeffient b in the research periods are about 0.96 and 0.97, respectively. Thus they are slightly below unity which contradicts with Gibrat's law. Note that error terms in the first regression equation (panel 1) do not conform to normality. This means that the probability levels for the T-statistic are incorrect. However, the test statistic is based on the sum of error terms so that, on the basis of the central limit theorem, the distribution of the statistic may anyway be close to normality. Hence the inference may provide us a good approximation. This same also holds for the OLS and correlation results below. Note that in this case it could be possible also to use for example the Dickey-Fuller distribution instead of t-distribution to improve the accury of the test.

The table also includes exemplary values for the four-year growth rate calculated for the percentiles of net sales. These exemplary values show that the growth is not very sensitive to size being, however, lower in larger firms. For example, the growth estimate for the period 1987–91 is about 41% for the 75% percentile (upper quartile) and about 49% for the 25% percentile (lower quartile). *Thus, we can conclude that there is a negative relationship between the growth and size but that the effect of size on growth is not very strong, except in*

TABLE 7. Regression results for logarithmic net sales (1991/1987 and 1995/1991)

Panel 1. Logarithmic sales in 1991 explained by logarthmic sales in 1987

Variable:	Parameter estimate	Standard error	T-statistic	Probability level
Intercept	0.6075	0.1385	4.387	0.0001
Logaritmic sales in 1987	0.9616	0.0215	44.765	0.0001

Adjusted R^2 = 0.9277 (F-value 2003.900 with probability 0.0001)

Jarque-Bera statistic on residuals = 83.7987 with probability 0.0001

Exemplary values for four-year growth estimate for alternative net sales values in 1987:

Percentile	95	75	50	25	5
Net sales	7969	1035	452	223	126
Growth estimate	29.99	40.59	45.14	49.14	52.44

Panel 2. Logarithmic sales in 1995 explained by logarthmic sales in 1991

Variable:	Parameter estimate	Standard error	T-statistic	Probability level
Intercept	0.2593	0.1730	1.499	0.1360
Logaritmic sales in 1991	0.9739	0.0254	38.304	0.0001

Adjusted R^2 = 0.9038 (F-value 1467.225 with probability 0.0001)

Jarque-Bera statistic on residuals = 4.8709 with probability 0.08756

Exemplary values for four-year growth estimate for alternative net sales values in 1991:

Percentile	95	75	50	25	5
Net sales	10599	1548	621	312	183
Growth estimate	1.75	6.99	9.57	11.56	13.12

very small and very large firms. The negative relationship also holds for the logarithmic growth and size. The Pearson correlation coefficients between these variables for the research periods were –14.2% (probability level 0.0755) and –8.2% (0.3063), respectively. These results may give weak support to Kalecki's model.

Table 8 presents the transition matrice for the growth rate from the period 1987–91 to the period 1991–95. This table evidently shows that the firms which grew slowly in the former period have better probabilities to grow fast in the latter period than the other firms. Thus there is obviously some negative persistency in growth processes. This interpretation is supported by the regression analysis results presented in panel 2 of the same table. There is a statistically significant negative relationship between the four-year growth rates. This result is almost identical with the one based on the correlation between the logarithms of successive growth rates. The Pearson coefficient of correlation for these logarithmic rates is –24.3% with a probability level 0.0022. *Thus, the results on the persistence of growth violate Gibrat's law*

TABLE 8. Relationship between the growth rates in 1987–91 and 1991–95

Panel 1. Markov transition probability matrice for the growth rate 1987–91

Growth class in 1987–1991:	Growth class in 1991–1995:							
	< 0	0–10	10–20	20–30	30–40	40–50	50–	Total
< 0	33.33	38.10	19.05	9.52				100
0–10	34.43	39.34	16.39	8.20	1.64			100
10–20	45.16	40.32	11.29	1.61		1.61		100
20–30	62.50	37.50						100
30–40	66.67	33.33						100
40–50								
50–	100.00							100

Panel 2. Regression model of growth rate 1991–1995

Variable:	Parameter estimate	Standard error	T-statistic	Probability level
Intercept	5.1830	1.1486	4.513	0.0001
Growth rate in 1987–1991	–0.2549	0.0826	–3.085	0.0024

Adjusted R^2 = 0.0518 (F-value 9.517 with probability 0.0024)

Jarque-Bera statistic on residuals = 27.6918 with probability 0.0001

and may give some empirical support to Ijiri-Simon's model (although no birth-and-death process is allowed).

The final point to be considered in this study is the stochasticity of growth. This point will be roughly evaluated by analysing the relationship of growth to logarithmic size, profitability and solidity. Table 9 presents the results of the regression analysis in which the average annual growth rate is explained by the logarithmic size, the return on investment ratio, and the debt-to-assets ratio. There is no statistically significant linear dependence of growth to the ratios since the coefficients of multiple determation are very low for both research periods. Table 10 presents some statistics about the financial ratios by growth class. For the former period (1987–91) the firms in the highest growth classes also show the highest profitability and solidity (lowest debt-to-assets) figures. However, in the latter period (1991–95) the results are reversed: fast growing firms tend to show lower profitability and solidity ratios than the others (except the highest growth class with one observation). *Thus, while supporting, on the one hand, the stochasticity of growth, there seems, on the other hand, to be evidence of a dependence of growth on some financial ratios.* This dependence may hold only for a small part of the population (a few of the largest firms) so that there is no statistically significant (linear) dependence.

TABLE 9. Regression model of growth rate in 1987–1991 and 1991–1995

Panel 1. Annual growth rate in 1987–1991

Variable:	Parameter estimate	Standard error	T-statistic	Probability level
Intercept	21.0480	5.5959	3.761	0.0002
Logaritmic sales in 1987	–1.3114	0.6392	–2.052	0.0419
Return on investment in 1987	–0.0166	0.0348	–0.478	0.6332
Debt-to-assets ratio in 1987	–0.0407	0.0497	–0.819	0.4139

Adjusted R^2 = 0.0098

Jarque-Bera statistic on residuals = 348.982 with probability 0.0001

Panel 2. Annual growth rate in 1991–1995

Variable:	Parameter estimate	Standard error	T-statistic	Probability level
Intercept	3.1234	6.0111	0.520	0.6041
Logaritmic sales in 1987	–0.5977	0.6660	–0.897	0.3709
Return on investment in 1991	0.0094	0.1614	0.058	0.9538
Debt-to-assets ratio in 1991	0.0546	0.0548	0.997	0.3205

Adjusted R^2 = –0.0065

Jarque-Bera statistic on residuals = 16.8138 with probability 0.0002

5. SUMMARY OF THE RESULTS

The purpose of the study was to analyse the growth processes, as well as the relationship between size and growth, in large Finnish firms in 1987–1995. This time period is very interesting because of radical changes in business cycles in that time. The study was concentrated on growth as a stochastic process and largely rested on the test of Gibrat's law of proportionate effect, which tells that growth is a random process and independent of the size of the firm. If Gibrat's law holds, there is no optimum size for the firm with respect to growth. If the growth of firms does not include the birth-and-death process, Gibrat's law generates a lognormal distribution and the variance of logarithmic size will continuously increase in time (pure Gibrat's law). However, if there is a linear negative dependence between the logarithms of growth and size, Gibrat's law will generate a lognormal distribution but with a constant variance of logarithmic size (Kalecki's model). If the mathematical expectation of increments in the stochastic transition matrix is negative, Gibrat's law generates a Pareto distribution (Champernowne's model). The incorporation of the birth-and-death process in Gibrat's law leads to a Yule distribution when there is a constant rate of birth (Simon's model) or serial correlation between

TABLE 10. Return on investment ratio and debt-to-assets ratio by growth class

Panel 1. Annual growth rate in 1987–1991

Growth class in 1987:	Return on investment ratio in 1987		Debt-to-assets ratio in 1987		
	Mean	Median	Mean	Median	Frequency
< 0	–0.22	2.56	61.83	63.37	21
0–10	2.99	1.59	66.65	68.93	61
10–20	–1.88	1.70	67.85	69.31	62
20–30	3.61	1.38	70.63	66.50	8
30–40	5.62	4.52	52.67	59.07	3
40–50					0
50–	5.56	5.56	51.23	51.23	2

Panel 2. Annual growth rate in 1991–1995

Growth class in 1987:	Return on investment ratio in 1987		Debt-to-assets ratio in 1987		
	Mean	Median	Mean	Median	Frequency
< 0	0.59	0.85	63.55	61.22	65
0–10	2.19	2.27	61.24	60.87	61
10–20	0.16	0.35	71.23	73.33	21
20–30	–2.56	0.40	73.46	73.66	8
30–40	–1.02	–1.02	73.79	73.79	1
40–50	3.00	3.00	70.92	70.92	1
50–					0

periodic growth rates (Ijiri-Simon's model). These properties of stochastic models gave an interesting basis for the empirical study.

The data for the present study were taken from the ETLA data base including financial statements from the 500 largest firms in Finland. However, it was required that the financial statements for the firm to be chosen had to be available for the entire period of 1987–95. All the firms that were involved in large mergers during the years 1987–1995 were excluded from the sample. These restrictions meant that only 157 firms out of 500 were accepted for the sample. The research period was split into two sub-periods, 1987–1991 and 1991–1995. The size of the firm was measured by nominal net sales. The distribution of the sample firms among industries was very diversified. Moreover, the data consisted of large and medium-sized Finnish firms with wide variation in size. The size distributions and growth processes were analysed by a number of methods. The most important methods were Markov transition matrices and regression analysis. Several measures of concentration were also applied (Hirschman-Herfindahl index, Gini coefficient, and concentration ratios) to analyse changes in the size distributions.

In general, the empirical results showed that the growth processes in large Finnish firms in 1987–1995 have operated in such a way that the concentration of net sales has been extremely stable. There are practically speaking no changes in concentration in the research period in spite of the radical changes in business cycles. The growth processes were almost similar in both sub-periods except that the growth was slower in the latter period due to the recession. There were no large differences in growth distribution between size classes. However, in the largest size class the growth seemed to be slower than in smaller classes. Moreover, the firms in the smallest size class tend to grow rather fast. *These two phenomena contradicted with Gibrat's law and may have made it possible for concentration to stay stable over time.* Consequently, there was observed a negative relationship between the growth and size but that the effect of size on growth was not very strong, except in very small and very large firms (compare with Kalecki's assumption). There was also a statistically significant negative relationship between the four-year growth rates. Thus, the results on the persistence of growth violated Gibrat's law and gave support to Ijiri-Simon's model. Finally, the results, on the one hand supported the stochasticity of growth but, on the other hand, gave also evidence of a dependence of growth on some financial ratios. This dependence may hold only for a small part of the population (a few of the largest firms) so that there is no statistically significant (linear) dependence. ■

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APPENDIX 1. Cumulative sales distributions

Panel 1. Sales distribution in 1987

Upper limit:	Size class:	Cumulative distribution:	Lognormal distribution:	Normal distribution:	Pareto coefficient:
200	1	2.08	20.76	35.07	
400	2	6.61	39.59	37.16	1.90
800	3	15.56	61.28	41.44	1.86
1600	4	23.09	79.88	50.26	2.22
3200	5	31.63	91.75	67.44	3.01
6400	6	47.64	97.37	91.04	4.34
12800	7	72.72	99.36	99.91	5.78
25600	8	100.00	99.88	100.00	
51200	9		99.98	100.00	
	10		100.00		

Panel 2. Sales distribution in 1991

Upper limit:	Size class:	Cumulative distribution:	Lognormal distribution:	Normal distribution:	Pareto coefficient:
200	1	0.80	13.43	34.72	
400	2	3.67	28.96	36.14	
800	3	10.53	49.89	39.04	1.90
1600	4	18.49	70.85	45.01	1.97
3200	5	26.21	86.45	57.14	2.61
6400	6	40.65	95.08	78.55	3.61
12800	7	64.31	98.62	97.79	4.98
25600	8	78.81	99.71	100.00	12.32
51200	9	100.00	99.95	100.00	
	10		100.00		

Panel 3. Sales distribution in 1995

Upper limit:	Size class:	Cumulative distribution:	Lognormal distribution:	Normal distribution:	Pareto coefficient:
200	1	0.57	12.58	34.44	
400	2	3.52	27.17	35.71	
800	3	8.85	47.26	38.29	1.84
1600	4	16.08	68.09	43.61	2.02
3200	5	24.61	84.35	54.50	2.54
6400	6	37.57	93.92	74.56	3.16
12800	7	59.26	98.15	96.05	3.98
25600	8	73.46	99.57	100.00	14.06
51200	9	100.00	99.92	100.00	
	10		100.00		